# Nonminimal Derivative Couplings and Inflation in Generalized Theories of Gravity

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**Abstract**

We study extended theories of gravity where nonminimal derivative couplings of the form $R^{kl}\phi_{,k}\phi_{,l}$ are present in the Lagrangian. We show how and why the other couplings of similar structure may be ruled out and then deduce the field equations and the related cosmological models. Finally, we get inflationary solutions which do follow neither from any effective scalar field potential nor from a cosmological constant introduced "by hand", and we show the de Sitter space–time to be an attractor solution.

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# 1 Introduction

The existence of an inflationary phase in the early universe is now generally accepted, cf. e.g. [1]. The first idea how to generate an inflationary de Sitter phase was to introduce a cosmological term $\Lambda > 0$. However, this idea suffered from the cosmological constant problem: In natural units, one gets $\Lambda \simeq 10^{-128}$, an almost unexplainable fine–tuning would be necessary to achieve that.

One of the next ideas to get an inflationary phase was to introduce other fields or other gravitational field equations. For instance, the Starobinsky model has a gravitational Lagrangian[1]

$$L = -\frac{R}{2} + \frac{l^2}{12} R^2 \,, \tag{1.1}$$

where $l$ is a constant of length–dimension and we have no cosmological constant.

In [2] it was shown, that, nevertheless, the de Sitter spacetime with the effective value of $\Lambda$ depending on $l$ is a transient attractor of the corresponding fourth–order vacuum field equations, cf.[3] for further details and references.

Soon it became clear how this behavior can be explained: By a conformal transformation (see e.g.[4][2]) which has an almost constant conformal factor near the de Sitter spacetime, the models with $L = f(R)$ (where $f$ is nonlinear in $R$), can be transformed to Einstein's theory with a minimally coupled scalar field $\phi$ and a potential $V(\phi)$ describing its self–interaction. In regions of an almost constant positive potential $V$, we can interpret $V(\phi)$ as an effective cosmological constant leading to a quasi de Sitter phase.

On another branch of research, also nonminimally coupled scalar fields have been used to deduce the inflationary phase, i.e.

$$L = F(\phi, R), \quad \text{with} \quad \frac{\partial^2 F}{\partial \phi \partial R} \neq 0 \,, \tag{1.2}$$

i.e., this Lagrangian does not have the form of $L = f(R) + V(\phi)$. However, also this kind of theories is conformally related (up to singular exceptions) to the theories mentioned above, but they deserve a lot of consideration since they allow, several times, to get inflation without the "graceful exit" problem bypassing the shortcomings of former inflationary models (see e.g. [5] for extended and hyperextended inflation).

However, the form of nonminimal coupling besides higher–order terms in the effective gravitational Lagrangian can be chosen in several ways (see e.g. [8]) to obtain one or more than one inflationary phases but the ingredients, after a conformal transformation are always the same: Inflation is driven by a scalar field potential which, for a certain period, assumes the appearance of an effective cosmological constant.

[1]We choose sign conventions such that the de Sitter spacetime has a curvature scalar $R < 0$, and the +-sign in Eq.(1.1) shows that we restrict to the tachyonic–free case.

[2]This conformal relation was independently found by several authors; it should be called Bicknell-theorem.

Our issue is now: Is it possible to recover the cosmological constant, and then the inflationary phase "without" considering any effective potential? In [6] and [7], it was discussed how to construct an effective cosmological constant starting from extended gravity theories (i.e. nonminimally coupled or higher–order theories). As the main result, an extension of the cosmic no hair conjecture was found. In any case, the scalar field potential in nonminimally coupled theories or the conformally related scalar field potential for higher–order theories were essential features. In [6], it was shown that for nonminimally coupled theories without a scalar field potential (e.g. a pure Brans–Dicke theory) an effective cosmological constant is never recovered.

In spite of this result, in [10], it was shown that an effective cosmological constant can be recovered if a nonminimal derivative coupling is introduced in the gravitational Lagrangian also if no scalar field potential or higher–order terms in curvature invariants are taken into account. In other words, it seems that a new type of inflation can be dynamically induced just by considering the self–coupling between geometry and the kinetic term of some given scalar field.

In 1993, Amendola [9] started to consider further types of coupling between curvature and the scalar field, called nonminimal derivative coupling, see also [10] for details. The main ingredient of this kind of couplings, already mentioned in [1], Eq.(9.5.9), reads

$$L_1 = R^{kl}\phi_{,k}\phi_{,l}\,. \tag{1.3}$$

The aim of the present paper is to study this kind of couplings and connect them with inflation.

The paper is organized as follows: In Sect. 2, we consider all Lagrangians of type (1.3) and find out which of them are really independent. In Sect. 3, the field equations are deduced; Sect. 4 deals with the corresponding cosmological models. Discussion and conclusions are drawn in Sect. 5. In particular, we discuss the results in relation to the transformations

$$\tilde{g}_{ij} = \frac{\partial \mathcal{L}}{\partial R_{ij}}\,, \tag{1.4}$$

see [11] and

$$\hat{g}_{ij} = g_{ij} + \lambda^2 u_i u_j\,. \tag{1.5}$$

see [12] asking for a generalization of conformal transformations in which it is possible to find out the analogous ingredients of a scalar field potential.

# 2 The Possible Lagrangians

The following six terms carry a geometric structure similar to $L_1$, Eq. (1.3):

$$\begin{aligned} L_2 &= R\phi_{,k}\phi^{,k}\,, & (2.1)\\ L_3 &= R\phi\Box\phi\,, & (2.2)\\ L_4 &= R^2\phi^2\,, & (2.3) \end{aligned}$$

$$L_5 = R_{,k}\phi^{,k}\phi\,, \tag{2.4}$$
$$L_6 = R^{kl}\phi\phi_{;kl}\,, \tag{2.5}$$
$$L_7 = \phi^2\Box R\,. \tag{2.6}$$

This list is complete in the sense that every scalar of the same geometric structure can be written as linear combination of $L_1, \ldots, L_7$[3]. To find out an independent subset of $L_1, \ldots, L_7$, we apply the fact that the addition of a divergence does not alter the field equation and is therefore not necessary for our purposes.

Using the divergencies

$$(R\phi^{,k}\phi)_{;k}\,, \quad (R^{ik}\phi\phi_{,k})_{;i}\,, \quad (R^{,i}\phi^2)_{;i}\,,$$

we conclude that without loss of generality, $L_5$, $L_6$ and $L_7$ are not necessary to be considered. Though $\Box(R\phi^2)$ represents a divergence of the same structure, it does not further reduce the necessary set $\{L_1, \ldots, L_4\}$. $L_4$ may be ruled out because it has already the structure of Eq. (1.2). Further, $L_3$ is only marginally interesting here, because it contains also $\phi$ itself, and we are mainly interested in a coupling, where only the gradient of $\phi$ is included.

Therefore, our main topic is to consider $\mathcal{L}_1$ and $\mathcal{L}_2$ (and sometime $\mathcal{L}_3$, too). That these three Lagrangians are really independent will become clear after having deduced the field equations.

# 3 How to Deduce the Field Equations

In subsection 3.1 we apply the variational derivative $\delta/\delta\phi$ and in 3.2 analogously $\delta/\delta g_{ij}$ to the Lagrangian density $\mathcal{L}_i = \sqrt{-g}L_i$, $i = 1, 2, 3$, cf. Eqs. (1.1), (2.1), (2.2).

## 3.1 Field Equation for the Scalar Field

From $\mathcal{L}_3$ we get

$$0 = 2R\Box\phi + \phi\Box R + 2R_{,k}\phi^{,k} \tag{3.1}$$

which can be written in a more compact form as

$$0 = R\Box\phi + \Box(R\phi).$$

For $\mathcal{L}_1$ and $\mathcal{L}_2$ we give a common deduction. To this end we define the tensor

$$V^{kl} = R^{kl} + \alpha R g^{kl}\,, \tag{3.2}$$

[3]This list is almost identical to the list Eq. (1.2) of Ref. [9]; the difference is the following: our $L_4 = R^2\phi^2$ is absent in [9], because there the appearance of derivatives was required, whereas we apply the more geometric point of view that $R^2$ and $\Box R$ have the same geometric structure, and so $L_4$ has to be included if we have $L_7 = \phi^2\Box R$. The fact that 3 of these terms may be neglected due to divergences, was deduced on another way in [9].

where $\alpha$ is a constant, and $L_0 = L_1 + \alpha L_2$, i.e.

$$L_0 = V^{kl}\phi_{,k}\phi_{,l}\,, \quad \mathcal{L}_0 = \sqrt{-g}L_0\,. \tag{3.3}$$

Using the formula

$$\frac{\partial \mathcal{L}_0}{\partial \phi_{,i}} = 2\sqrt{-g}V^{ik}\phi_{,k}\,,$$

we get the result that $0 = \delta\mathcal{L}_0/\delta\phi$ then reads

$$0 = -2(V^{ik}\phi_{,k})_{;i}\,. \tag{3.4}$$

From the contracted Bianchi identity, we get

$$V^{ik}{}_{;k} = \left(\frac{1}{2} + \alpha\right) R^{;i}$$

so $\alpha = -1/2$ plays a special role[4]. The field equation (3.4) for $\phi$ can be rewritten as (after dividing by -2)

$$0 = R^{ik}\phi_{;ik} + \alpha R\Box\phi + \left(\frac{1}{2} + \alpha\right) R^{;k}\phi_{,k}\,. \tag{3.5}$$

## 3.2 The Gravitational Field Equation

Let us start with the easier-to-deal case $L_2 = R\psi$, with

$$\psi = \phi_{,k}\phi_{,l}g^{kl}\,. \tag{3.6}$$

The field equation shall be deduced in 2 steps: First we consider the intermediately introduced auxiliary field $\psi$ as an independent scalar field (and that problem is of the known structure Eq. (1.2)), and second, we add the correction term $-R\phi^{,a}\phi^{,b}$ which results from the fact that $\psi$, Eq. (3.6), *has* a dependence on $g^{kl}$. As a result we get[5]

$$E_2^{ab} = \frac{1}{2}g^{ab}R\phi_{,k}\phi^{,k} - R\phi^{,a}\phi^{,b} - R^{ab}\phi_{,k}\phi^{,k} + (\phi_{,k}\phi^{,k})^{;ab} - g^{ab}\Box(\phi_{,k}\phi^{,k})\,, \tag{3.7}$$

where

$$E_i^{ab} = \frac{1}{\sqrt{-g}}\frac{\delta\mathcal{L}_i}{\delta g_{ab}}\,.$$

The same principle applies to $L_1 = R_{kl}u^{kl}$. First, $u^{kl}$ is considered as any contravariant symmetric tensor field, and second, the fact that $u^{kl} = \phi^{;k}\phi^{;l}$ depends on the metric (because the dependence of the full Lagrangian is on $\phi_{,k}$ and not on $\phi^{,k}$) via

$$u^{kl} = g^{ka}\phi_{;a}g^{lb}\phi_{;b}\,,$$

[4]This is the same case in Ref. [6, eq. (26)]

[5]Eq. (3.7) is identical to Eq. (3.4) of Ref. [9], which was given there without detailed explanation. Eq. (3.8), however, we did not find in the literature.

one has the correction terms $-R^{ak}\phi_{;k}\phi^{;b} - R^{bk}\phi_{;k}\phi^{;a}$. The final expression is this one

$$
\begin{aligned}
E_1^{ab} &= \frac{1}{2}g^{ab}R^{kl}\phi_{,k}\phi_{,l} - \frac{1}{2}\Box(\phi^{,a}\phi^{,b}) - \frac{1}{2}g^{ab}(\phi^{,k}\phi^{,l})_{;kl} \\
&\quad -[(\phi^{,k}\phi^{,(a})^{;b)}]_{;k} - 2R^{k(a}\phi_{,k}\phi^{,b)}\,,
\end{aligned}
\tag{3.8}
$$

where round brackets denote symmetrization. It should be noted that Eq. (3.8) has already a quite compact form. After multiplying out the derivatives one gets much more terms, and if one changes the ordering of the covariant derivatives one would produce extra terms like

$$\phi^{,c}\phi^{,d}R^{a\ b}_{\ c\ d}\,.$$

Both for Eqs. (3.7) and (3.8) the highest $\phi$–derivative is a third one.

# 4 Cosmological Models

In [10] the following Lagrangian $\mathcal{L} = \sqrt{-g}L$ has been discussed, where

$$L = -\frac{R}{2} + \frac{1}{2}g^{kl}\phi_{,k}\phi_{,l} + \zeta L_1 + \xi L_2\,, \tag{4.1}$$

where $L_1$ and $L_2$ are defined by Eqs. (1.3) and (2.1). From dimensional reason, $\zeta$ and $\xi$ have the dimension $l^2$, where $l$ is any length, $L_1$, $L_2$ represent corrections to the Lagrangian of Einstein's theory without the $\Lambda$–term, and with a minimally coupled scalar field without self–interaction.

For a spatially flat Friedman model

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \tag{4.2}$$

and

$$H = \frac{1}{a}\frac{da}{dt}\,,$$

we get with $\zeta = \xi = 0$

$$a(t) \sim t^{1/3} \quad \text{and} \quad \phi = \phi_c \ln t$$

where $\phi_c$ is an appropriately chosen constant. Thus, without the correction terms, no inflationary solution can be found. However, for $\zeta + 4\xi > 0$, an inflationary phase with

$$\Lambda = \frac{1}{2(\zeta + 4\xi)} \tag{4.3}$$

exists, see [10], Eq. (13). Let us now go into the details. In [10] the metric (4.2) was directly inserted into the Lagrangian (4.1). Here we first consider the field equation and insert the metric only afterwards. This has the advantage that the details will become more clear.

## 4.1 The Scalar Field Equation

For the Lagrangian $L$, Eq. (4.1), we get the field equation for $\phi$ as a linear combination of $\Box\phi$ (from the usual metric term in Eq. (4.1)) and Eq. (3.5), where now $\alpha$ has to be replaced by $\xi/\zeta$, and (3.5) has to be multiplied by $\zeta$. One can directly see:

- If $\phi_{,k}$ is covariantly constant and if $R$ is constant, then he equation is fulfilled.
- If $\phi_{,k}$ is covariantly constant and if $\alpha = -1/2$ (i.e. $\zeta = -2\xi$) then the equation is fulfilled.

Our main example is as follows: If $\phi = \phi_0 t$, where $\phi_0$ is any constant and $t$ is the time of metric (4.2), then

$$\phi_{;k} = (\phi_0, 0, 0, 0)\,, \quad \text{and} \quad \phi_{;kl} = -\Gamma^0_{kl}\phi_0\,.$$

So that we get $\Box\phi = 3H\phi_0$.

**Remark:** This implies that for $H\phi_0 \neq 0$, the vector $(\phi_0, 0, 0, 0)$ is not covariantly constant inspite of the constancy of $\phi_0$.

Now we assume $H$ to be a positive constant, i.e., metric (4.2) represent the de Sitter space–time[6]. To get a solution of the scalar field equation, we have either

- $\phi_0 = 0$;

or

- $\phi_0 \neq 0 \quad$ and $\quad 2\Lambda(\zeta + 4\xi) = 1$.

The latter case represents Eq. (4.3) above.

## 4.2 The Gravitational Field Equation

Now we insert this de Sitter solution $g_{ij}$ and $\phi = \phi_0 t$ into the gravitational field equation. The scalar $\psi$, Eq. (3.6) now reads

$$\psi = \phi_0^2 = const$$

From Eq. (3.7) we get

$$E_2^{ab} = 4\Lambda\phi^{,a}\phi^{,b} - \Lambda g^{ab}\phi_0^2$$

and a similar expression for $E_1^{ab}$. It turns out that for this highly symmetric case, the gravitational field equation does not give an additional condition, so the de Sitter space–time, with $\Lambda$ according to Eq. (4.3), is a solution.

[6]To simplify the comparison, we give here the known formalism for Einstein's theory: $\Lambda = 3H^2$, $R_{ij} = -\Lambda g_{ij}$, $R = -4\Lambda$, $R_{ij} - (R/2)g_{ij} = \Lambda g_{ij}$.

### 4.3 The Stability of the de Sitter solution

Let us now make the ansatz

$$a(t) = e^{\alpha(t)}\,, \tag{4.4}$$

where

$$\alpha(t) = \alpha_0 + H_0 t + \varepsilon \int \beta(t) dt\,, \quad \varepsilon \ll 1 \tag{4.5}$$

and

$$\varphi(t) = \varphi_1 + \varphi_0 t + \varepsilon \int \gamma(t) dt\,, \tag{4.6}$$

where

$$\dot{\varphi}(t) = \varphi_0 + \varepsilon\gamma(t)\,. \tag{4.7}$$

Here $\varphi_0$, $\varphi_1$, $\alpha_0$, $H_0$ are constants, the functions $\beta(t)$, $\gamma(t)$ have to be determined, and $\varepsilon$ is the parameter of linearization. From Eqs. (4.4)–(4.7) it follows that

$$H = \frac{\dot{a}}{a} = H_0 + \varepsilon\beta\,, \qquad \dot{H} = \varepsilon\dot{\beta}\,. \tag{4.8}$$

and the Eqs. of motion [eq. (9)–(11) of the paper [10]] assume the form

$$2\chi\varphi_0\varepsilon\ddot{\gamma} + 4\eta H_0\varepsilon\varphi_0\dot{\gamma} + \varepsilon\varphi_0(1+6H_0^2\eta)\gamma + 2\varepsilon(1+\eta\varphi_0^2)\dot{\beta} + 3H_0^2(1+\eta\varphi_0^2) + \frac{\varphi_0^2}{2} = 0 \tag{4.9}$$

$$6\chi\varepsilon\dot{\beta} - 12H_0(\eta - 3\chi)\varepsilon\beta + 18\chi H_0^2 - 6\eta H_0^2 + 1 = 0 \tag{4.10}$$

$$6\chi\varepsilon H_0\varphi_0\dot{\gamma} + \varphi_0\varepsilon(1+6H_0^2\eta)\gamma + 6H_0\varepsilon(1+\eta\varphi_0^2)\beta + 3H_0^2(1+\eta\varphi_0^2) + \frac{\varphi_0^2}{2} = 0 \tag{4.11}$$

where $\chi = -(2\xi+\zeta)$ and $\eta = -2(\xi+\zeta)$. The integration of (4.10) and (4.11) can be immediately carried out leading to the solutions

$$\beta(t) = \beta_0 e^{c_1 t} + c_2\,, \tag{4.12}$$

where $\beta_0$ is a constant[7],

$$c_1 = -\frac{2H_0(4\xi+\zeta)}{2\xi+\zeta}\,, \quad c_2 = \frac{1-6H_0^2(4\xi+\zeta)}{12H_0(4\xi+\zeta)}, \tag{4.13}$$

and

$$\gamma(t) = -\frac{a_1 a_3}{ba_1 + aa_2}\, e^{c_1 t} + \gamma_0 e^{-(a_2/a_1)t} - \frac{a_4}{a_2}\,, \tag{4.14}$$

where

$$a = 6\chi\varepsilon\,, \quad b = 12H_0(\eta - 3\chi)\varepsilon$$

$$a_1 = 6H_0\varphi_0\chi\varepsilon\,, \quad a_2 = (1+6H_0^2\eta)\varphi_0\varepsilon\,, \quad a_3 = 6(1+\eta\varphi_0^2)H_0\beta_0\varepsilon\,,$$

[7] From Eq. (5.1) it will become clear that in the region of parameters we are interested in both denominators of Eq. (4.13) remain positive numbers.

$$a_4 = 6H_0(1+\eta\varphi_0^2)\varepsilon c_2 + 3(1+\eta\varphi_0^2) + \frac{\varphi_0^2}{2}$$

We note that

$$\frac{a_2}{a_1} = \frac{12H_0^2(\xi+\zeta)-1}{6H_0^2(2\xi+\zeta)}\,. \tag{4.15}$$

Inserting (4.12) and (4.14) into (4.9) one gets a relation for these constants. As final comment, we infer the explicit expression of

$$\alpha(t) = (\alpha_0 + \varepsilon C_0) + (H_0 + c_2)t + \varepsilon\beta_0 c_1 e^{c_1 t} \tag{4.16}$$

and

$$\varphi(t) = \varphi_1 + \left(\varphi_0 - \varepsilon\frac{a_4}{a_2}\right)t - \varepsilon\frac{a_1 a_3}{(ba_1 + aa_2)c_1}e^{c_1 t} - \varepsilon\gamma_0\frac{a_1}{a_2}e^{-(a_2/a_1)t}\,. \tag{4.17}$$

The main point of the deduction is that $c_1$ from Eq. (4.13) is a negative real number.

Immediately we see that the conditions for the stability of the de Sitter are satisfied

$$\frac{\dot{H}}{H^2} \to 0\,, \quad \frac{\varphi(t)}{t} \to 0\,, \quad \text{as} \quad t \to \infty\,. \tag{4.18}$$

# 5 Conclusions

Which values of $\zeta$ and $\xi$ in eq. (4.1) will give sensible results? As it is always the case in such higher–derivative theories, several ranges of the parameters have to be excluded. We have already seen that for $\zeta = -2\xi$ we have a singular point of the differential equation and that we need $4\xi + \zeta > 0$ to ensure $\Lambda > 0$. So it seems to be adequate that we require $\zeta > -4\xi$ and $\zeta \neq -2\xi$. However, we will require a little stricter

$$\zeta > 4|\xi| \geq 0\,. \tag{5.1}$$

To discuss the stability of the de Sitter solution, one can compare it with other Friedman solutions; this has already been done in [10], where the de Sitter space–time has found to be a solution. (For ease of comparison: Eq. (5.1) implies $A < 0$ and $B > 0$ in [10], Eq. (16).) Then the field equation has solutions

$$H = c_1 \tanh(c_2 t) \quad \text{and} \quad H = c_1 \coth(c_2 t)\,,$$

$(c_1, c_2 > 0)$ both having $H \to const > 0$ as $t \to \infty$.

A more thorough discussion of the stability can be performed if the symmetry of the metric is not prescribed from the beginning. In order that the Cauchy problem be well–posed, we need some further conditions. (However, these conditions are fulfilled in a neighborhood of the interesting de Sitter space–time.)

The scalar field equation is of second order in $\phi$, so it has the structure

$$\phi_{,00} = F(\phi, \phi_{,0}, R_{ij}, R_{,0}) \tag{5.2}$$

(The dependency on $R_{,0}$ disappears for $\alpha = -1/2$, see Eq. (3.5), but this case is not covered by the allowed cases Eq. (5.1), and the dependency on the spatial derivative is not explicitly mentioned.) This equation (5.2) can be derivated with $d/dt$, and one gets

$$\phi_{,000} = G_1(\phi, \phi_{,0}, R_{ij}, R_{ij,0}, R_{,0}, R_{,00}) \tag{5.3}$$

The right–hand sides of Eqs. (5.2), (5.3) have to be inserted into the gravitational field equation to replace the artificial second and third derivative of $\phi$. Thus a fourth–order field equation for $g_{ij}$ results with leading–order term $R_{,00}$.

A more detailed elaboration of the corresponding stability has to be done yet. Further, let us mention that the terms $L_1$ and $L_2$ discussed above, can be related to the trace anomaly.

Finally, there seems strong evidence that the model discussed in this paper is not related by any conformal transformation to any known model. A further step for finding related models might be to rewrite the metric in a more general transformation of the form

$$\tilde{g}_{ab} = g_{ab} + \lambda R g_{ab} + \mu R_{ab} \tag{5.4}$$

with $\lambda$ and $\mu$ constants. (The Einstein tensor for $\tilde{g}_{ab}$ gives a tensor of order 4 if rewritten with $g_{ab}$). Another idea goes as follows: Following [11] (see the Introduction), we write (Eq. (4.1)

$$\tilde{g}^{ab} = \frac{\partial L}{\partial R_{ab}} = -\frac{1}{2} g^{ab} + \zeta \phi_{,a}\phi_{,b} + \xi g^{ab}\phi_{,k}\phi^{,k} \tag{5.5}$$

which gives a transformation also of a more general structure: It combines a conformal transformation with a Schild–transformation of type Eq. (1.5). It is not clear at the moment whether one of these transformations will simplify the equation or not. Probably we need an additional tensor field instead of an additional scalar field to be able to transform to Einstein's theory. The appearance of an inflationary solution in the nonminimal derivative coupling model discussed here can be explained as follows: In regions, where

$$R_{ij} \sim -\Lambda g_{ij} \qquad (\Lambda > 0)$$

and

$$\phi_{,k} \sim (\phi_0, 0, 0, 0), \quad (\phi_0 \neq 0)$$

the scalars $R^{ij}\phi_{,i}\phi_{,j}$ and $R\phi_{,k}\phi^{,k}$ both are negative and approximately constant. So, in this approximation, their appearance in the Lagrangian mimics an effective cosmological constant.


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